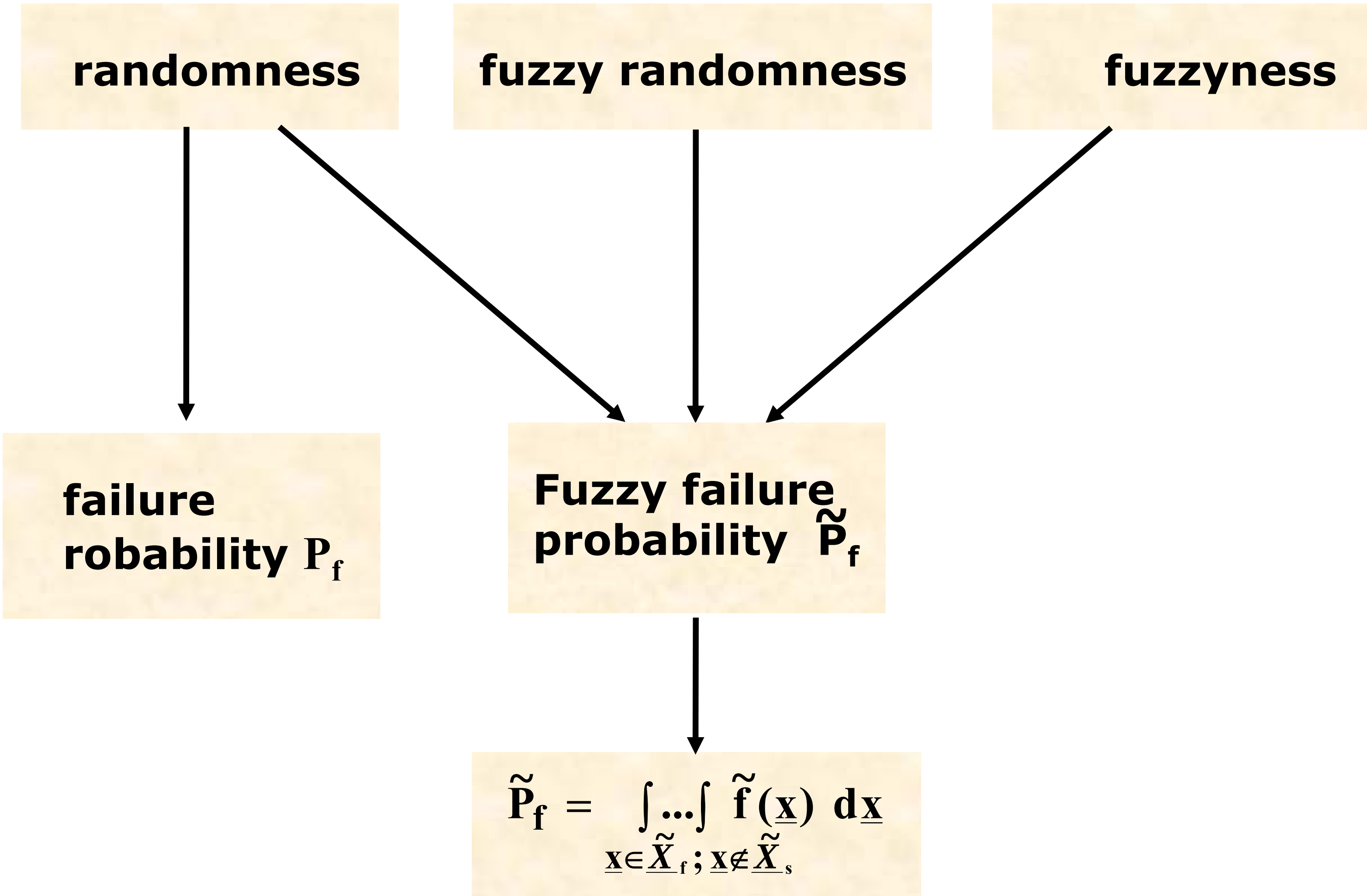
TECHNISCHE UNIVERSITÄT DRESDEN

Institute for Statics und Dynamics of Structures

Fuzzy probabilistic safety assessment

Bernd Möller

Fuzzy probabilistic safety assessment



Methods for computing the fuzzy failure probability $\tilde{P}_f$

- **Computing of $\tilde{P}_f$ original by original.**
  If one original from each fuzzy probability basic variable is known, the assigned failure probability may be calculated, e.g., by approximated integration.

- **Fuzzy-Monte-Carlo Simulation**

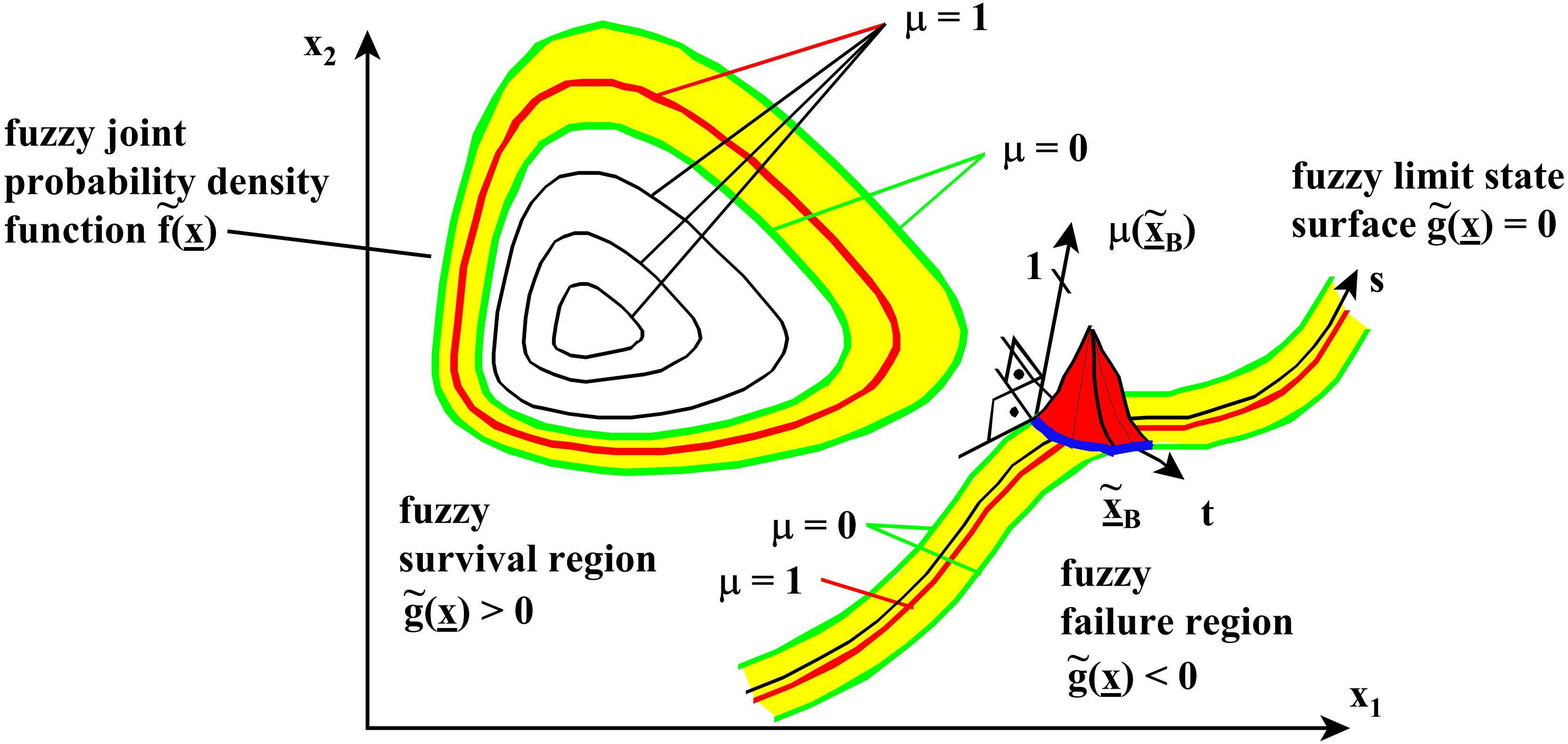
  Improving the numerical efficiency by:
  - Importance Sampling
  - Subset Sampling
  - Line Sampling

- **Fuzzy first order reliability - FORM**

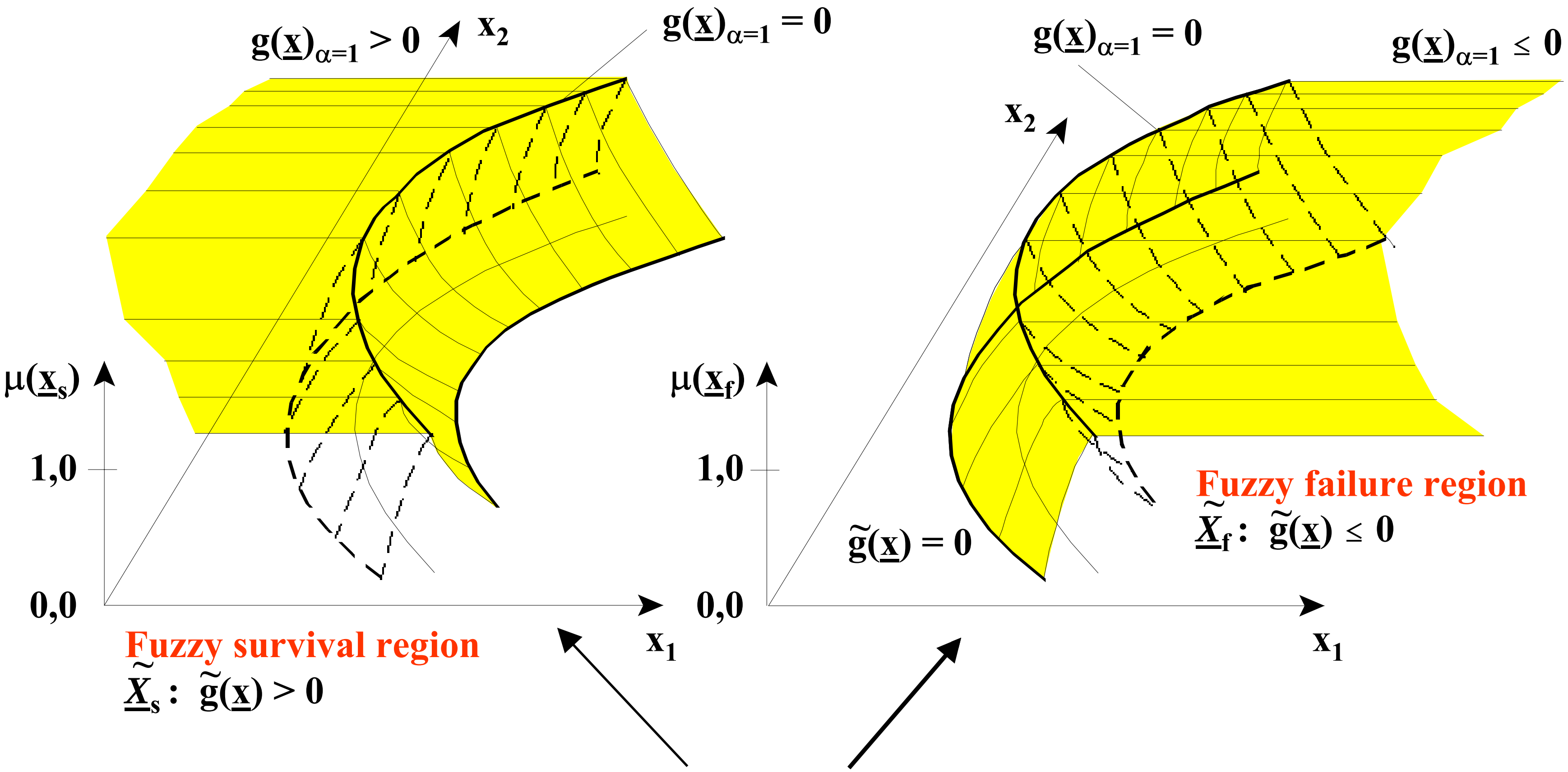
Original space of the basic variables

- data uncertainty $\Rightarrow \tilde{f}(\underline{x})$
- model uncertainty $\Rightarrow \tilde{g}(\underline{x}) = 0$

- **fuzzy design point $\underline{\tilde{x}}_B$**



Fuzzy first order reliability method

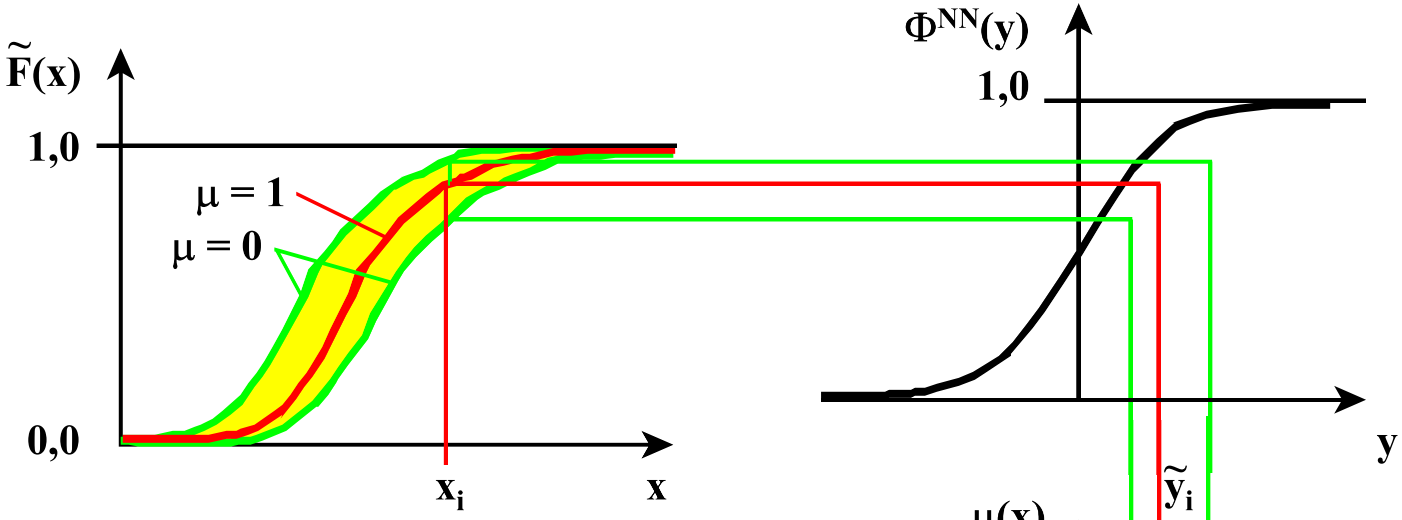


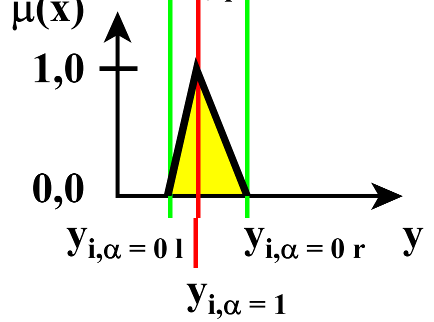
Fuzzy function

Fuzzy first order reliability method

Transformation of fuzzy random variables

- Space of the arbitrarily distributed random variable
- Standard normal space

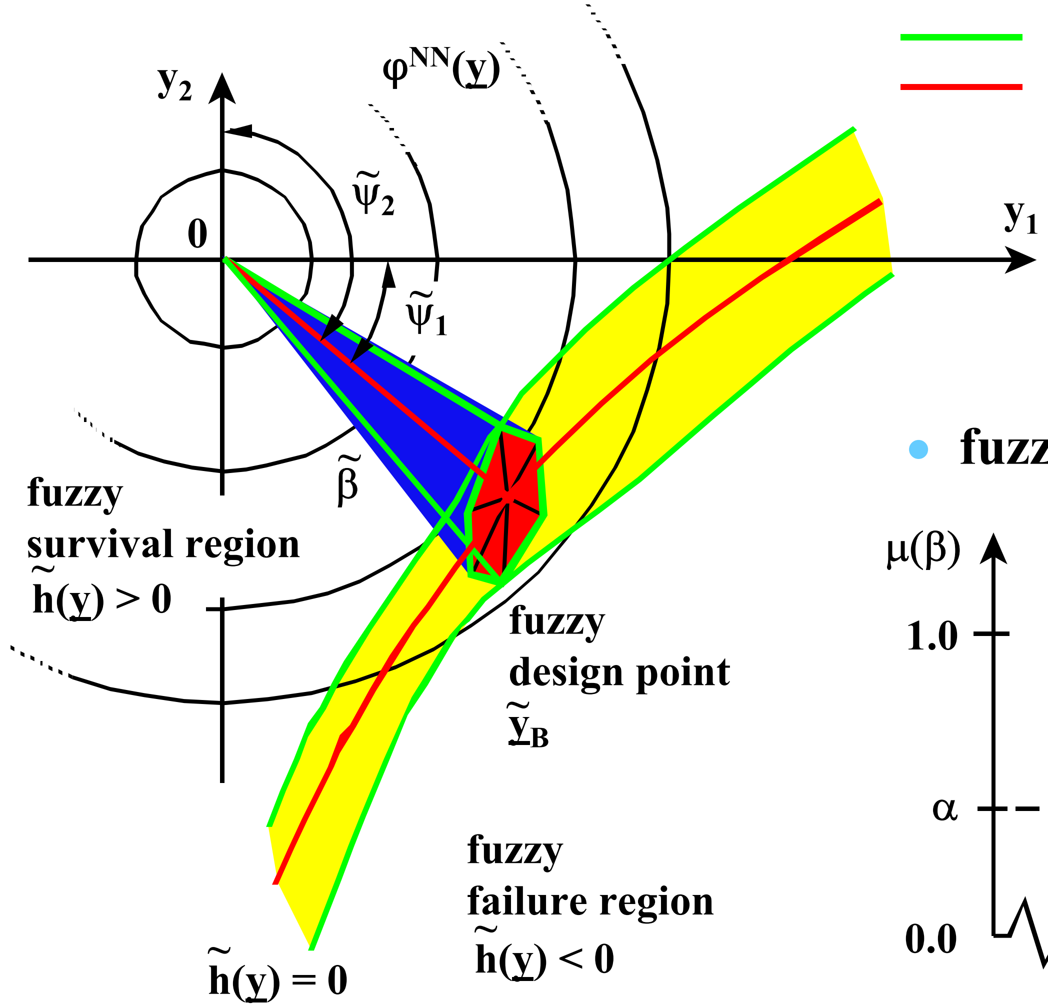




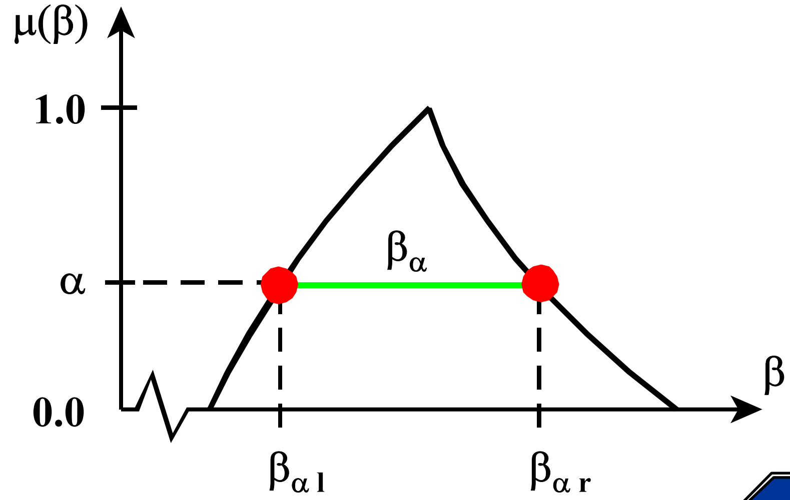
$$\tilde{y} = \Phi^{NN^{-1}}\left(\tilde{F}(x)\right)$$

Standard normal space

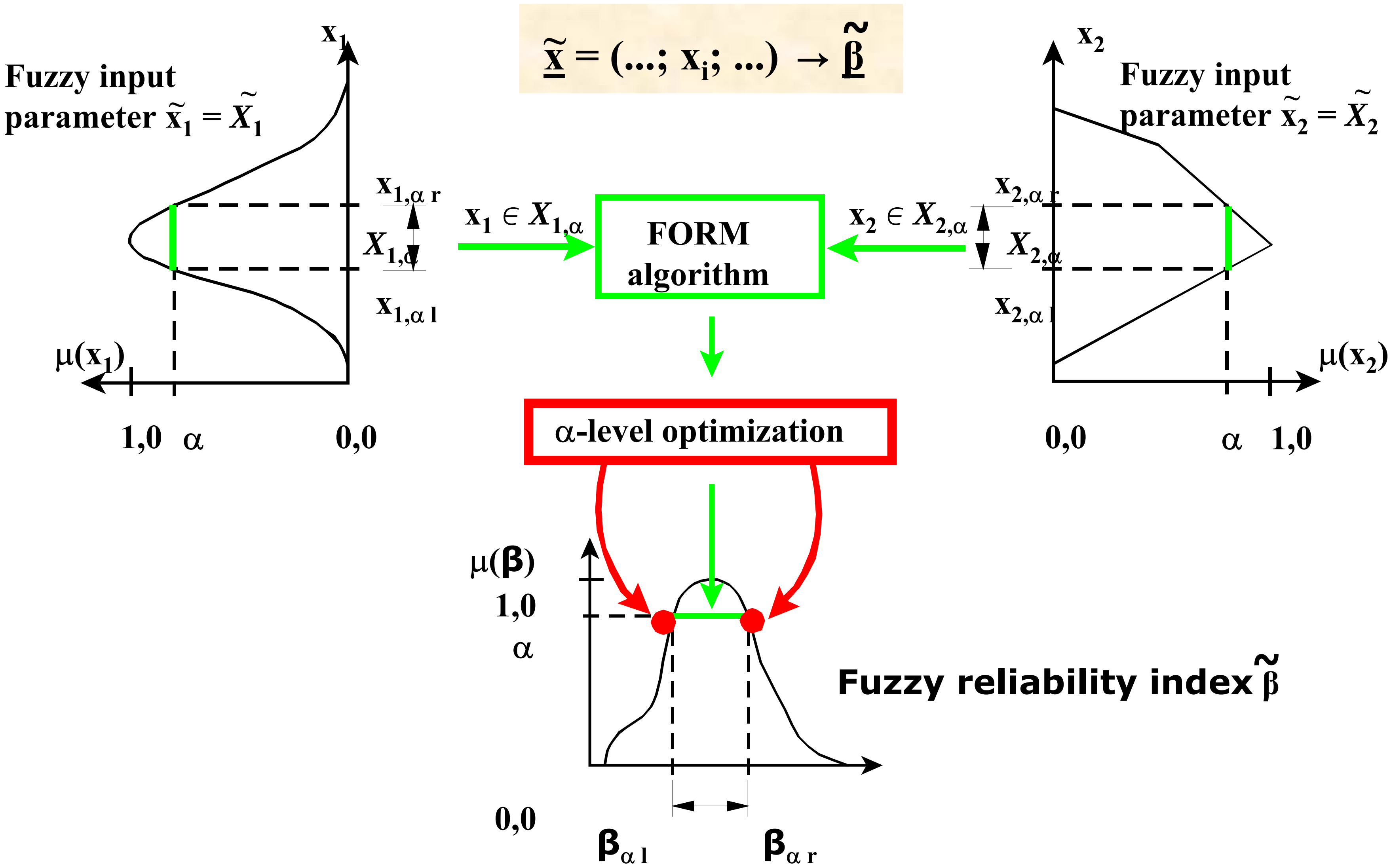
- complete fuzziness in $\tilde{h}(\underline{y}) = 0$ $\Rightarrow$ fuzzy design point $\tilde{\underline{y}}_B$



- fuzzy reliability index $\tilde{\beta}$

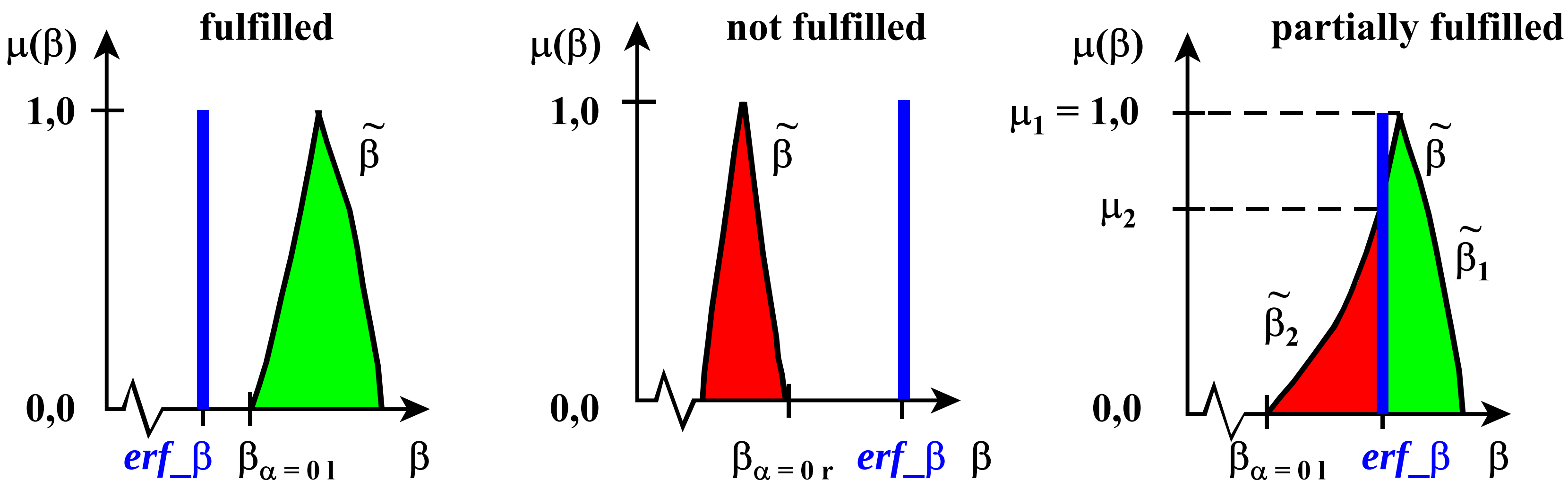


FFORM – numerical realization



Fuzzy first order reliability method

Safety verification: $\tilde{\beta} \geq erf\_\beta$



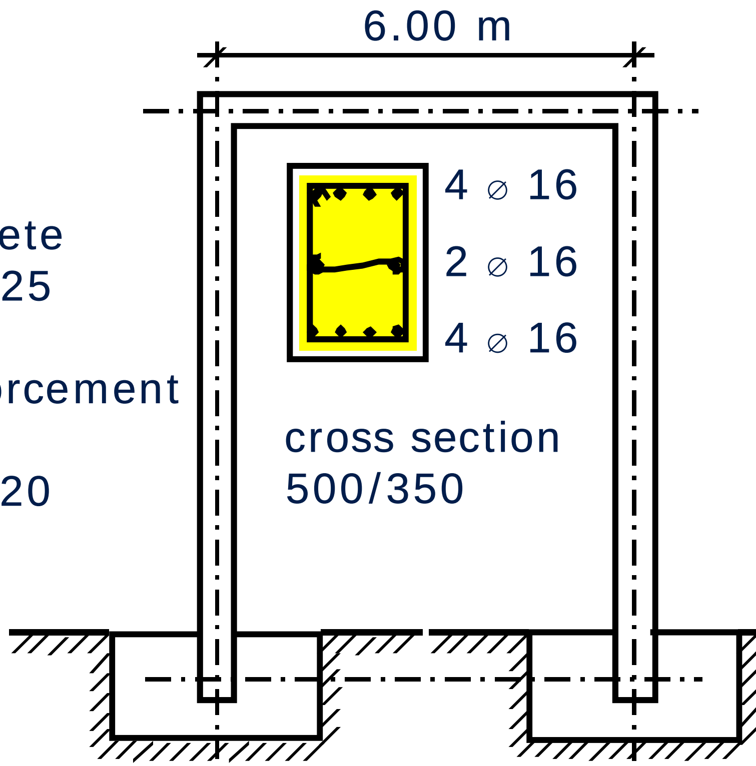
Example: Fuzzy stochatic safety assessment

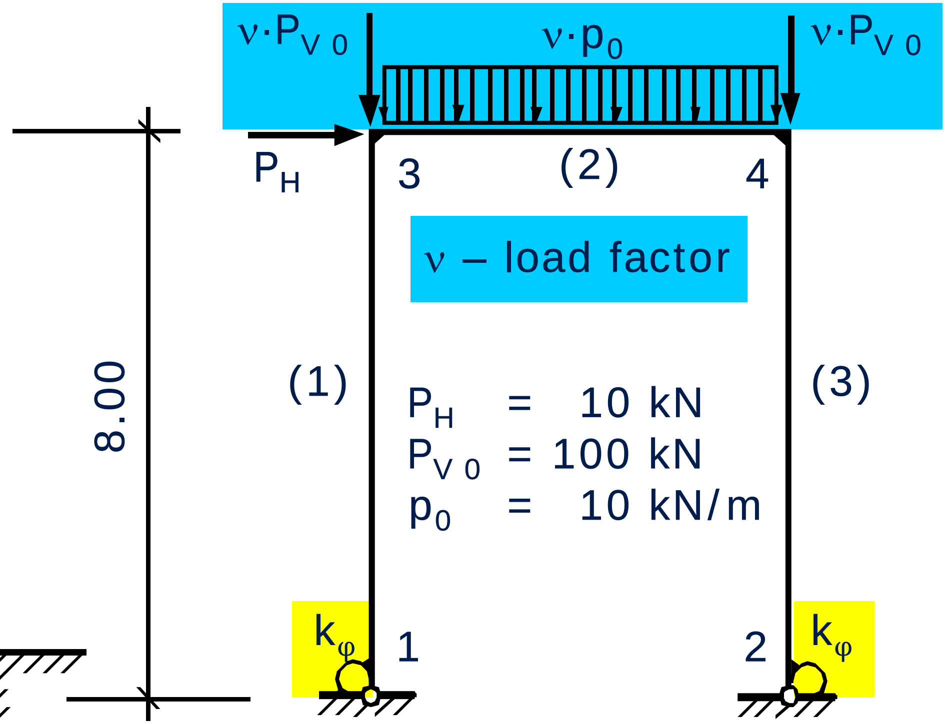
Example: Fuzzy stochastic safety assessment

Reinforced Concrete Frame

concrete
C 20/25

reinforcement
steel
BSt 420





Loading Process

- dead load
- horizontal load P_H
- vertical loads $\nu \cdot P_{V\,0}$ and $\nu \cdot p_{V\,0}$

fuzzy stochastic
basic variable

Features of the deterministic fundamental solution

- plane structural model with
  imperfect straight bars and layered cross sections

- numerical integration of the system of
  1st order differential equations for the bars

- interaction of internal forces

- incremental-iterative solution technique under
  consideration of complex loading processes

- consideration of all essential
  geometrical and physical nonlinearities

  - large displacements and moderate rotations
  - realistic material description of reinforced
    concrete including cyclic and damage effects

FFORM - Analysis I

fuzzy probabilistic basic variables

- **load factor ν (x_1)**

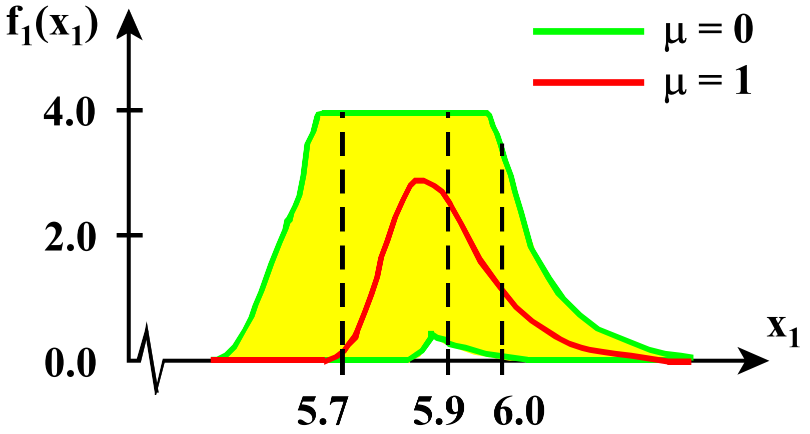
  extreme value distribution ex-max-type I (GUMBEL)

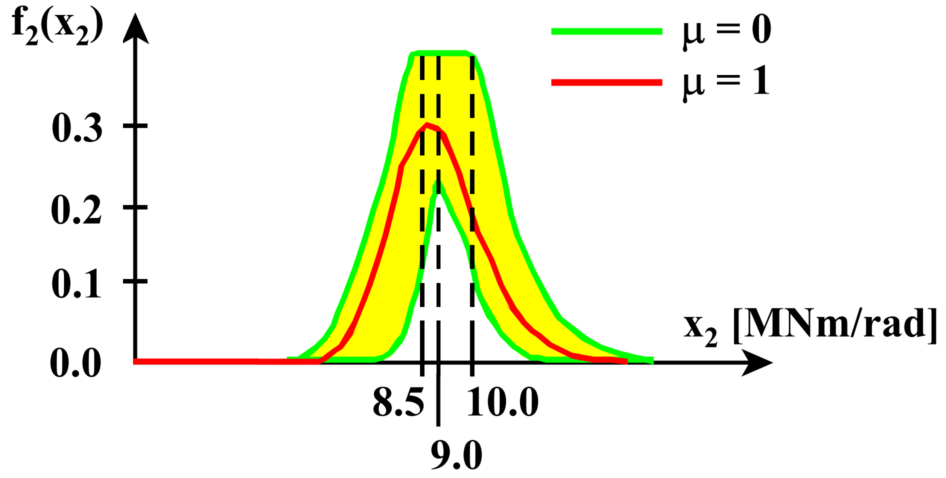
  $\tilde{m}_{x_1} = < 5.7; 5.9; 6.0 >$
  $\tilde{\sigma}_{x_1} = < 0.08; 0.11; 0.12 >$

- **rotational spring stiffness k_φ (x_2)**

  logarithmic normal distribution

  $x_{0,2} = 0$ MNm/rad
  $\tilde{m}_{x_2} = < 8.5; 9.0; 10.0 >$ MNm/rad
  $\tilde{\sigma}_{x_2} = < 1.00; 1.35; 1.50 >$ MNm/rad

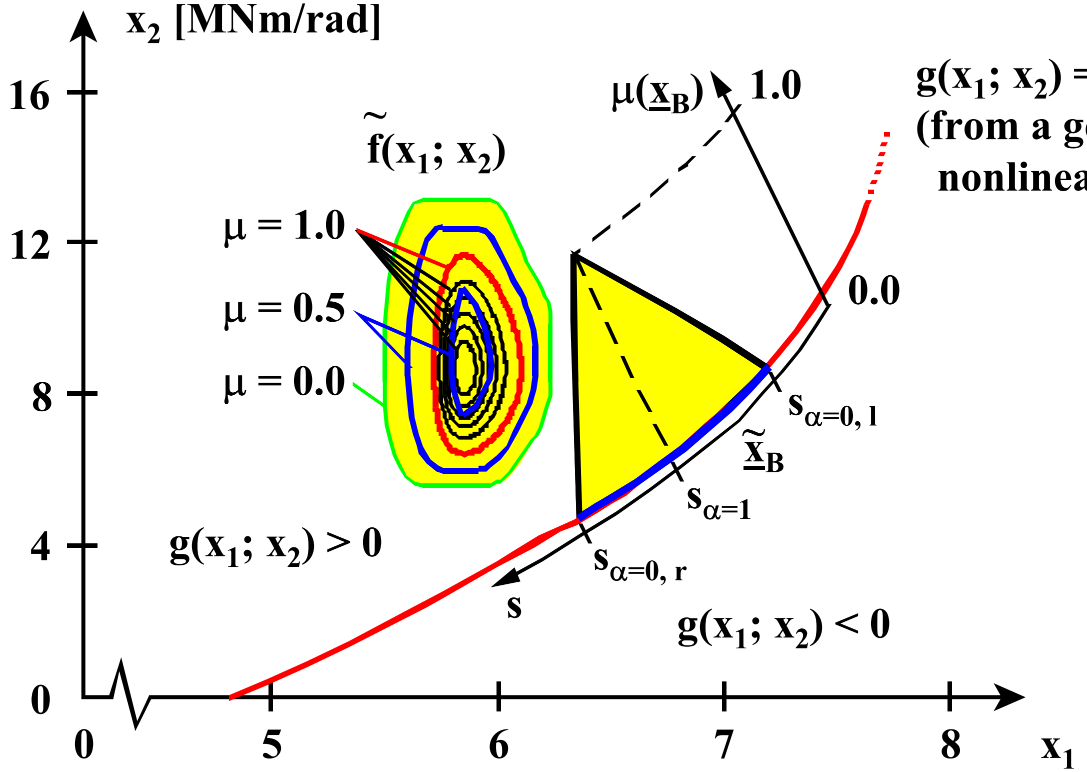




FFORM - Analysis I (cont'd)

original space of the fuzzy probabilistic basic variables

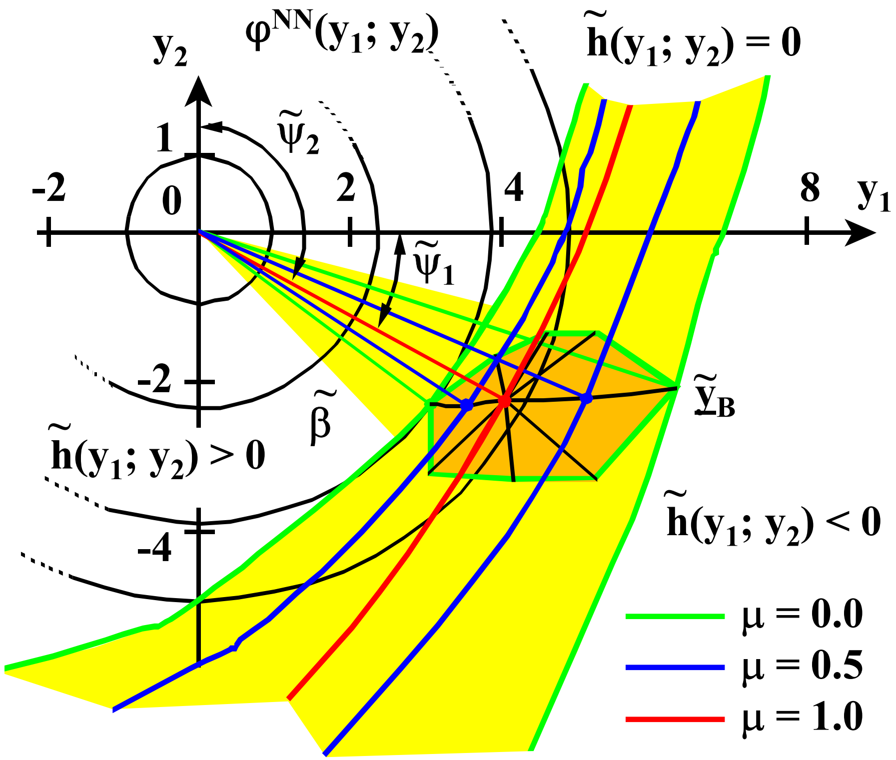
- **fuzzy joint probability density function, crisp limit state surface and fuzzy design point**



FFORM - Analysis I (cont'd)

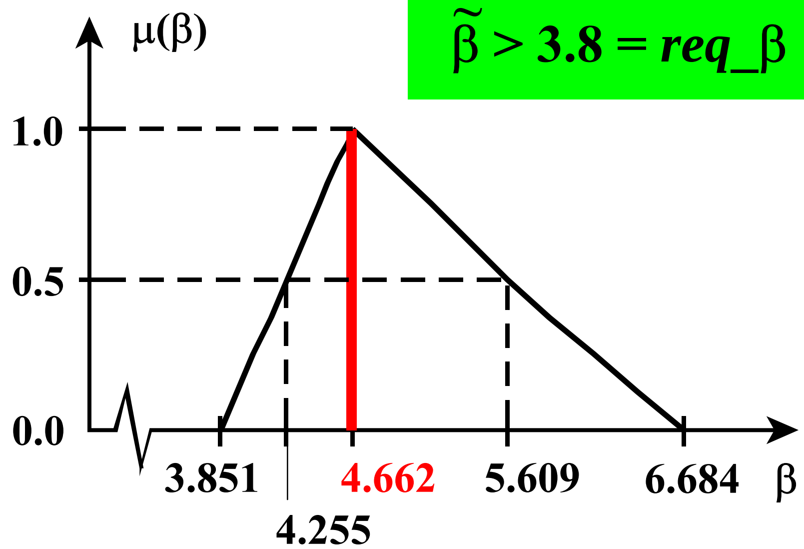
standard normal space

- **crisp standard joint probability density function, fuzzy limit state surface and fuzzy design point**



- **fuzzy reliability index, safety verification**

$\tilde{\beta} > 3.8 = req\_\beta$



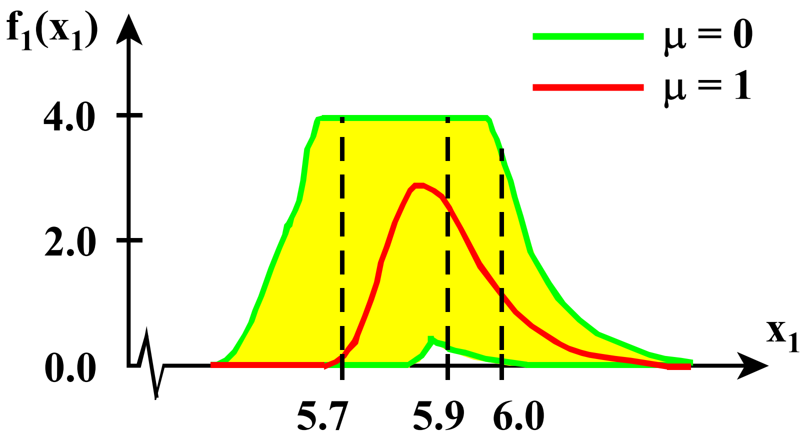
FFORM - Analysis II

data and model uncertainty

- **fuzzy probabilistic basic variable - load factor ν**

  extreme value distribution ex-max-type I (GUMBEL)

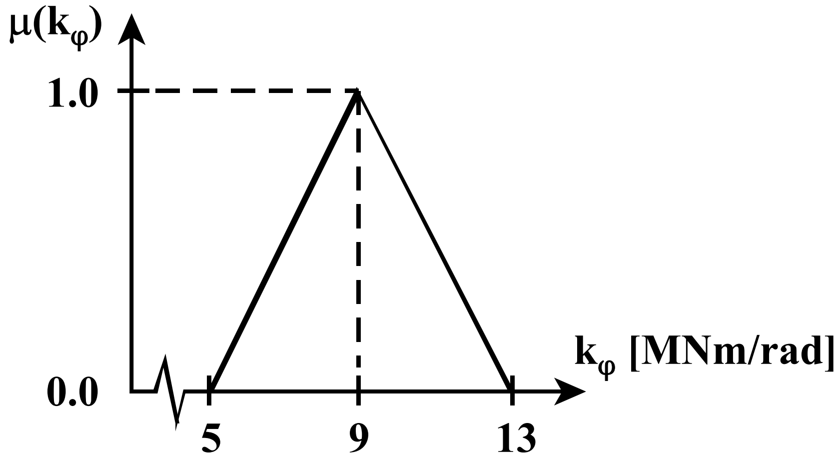
  $\tilde{m}_{x_1} = <5.7;\ 5.9;\ 6.0>$
  $\tilde{\sigma}_{x_1} = <0.08;\ 0.11;\ 0.12>$



- **fuzzy model parameter rotational spring stiffness $\tilde{k}_\varphi$**

  fuzzy triangular number

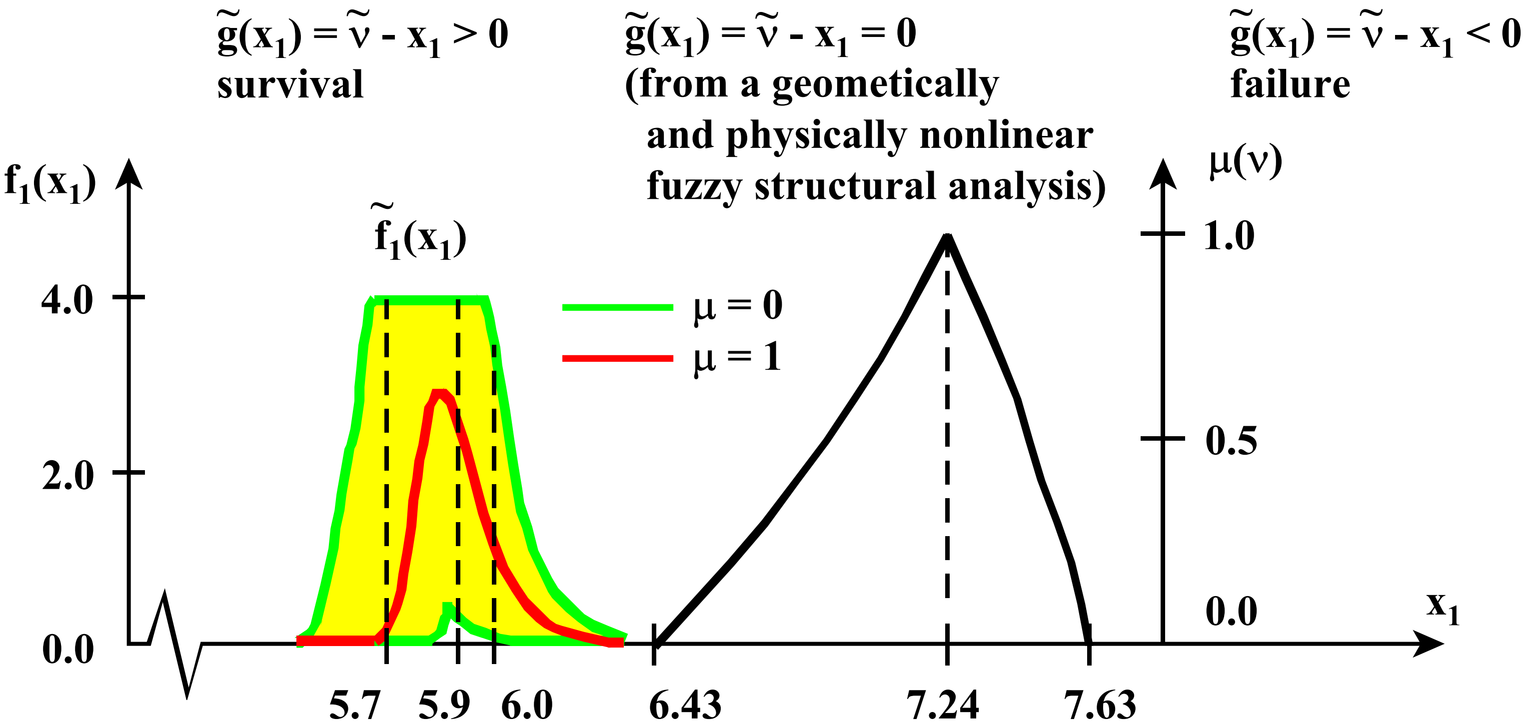
  $\tilde{k}_\varphi = <5;\ 9;\ 13>$ MNm/rad



FFORM - Analysis II (cont'd)

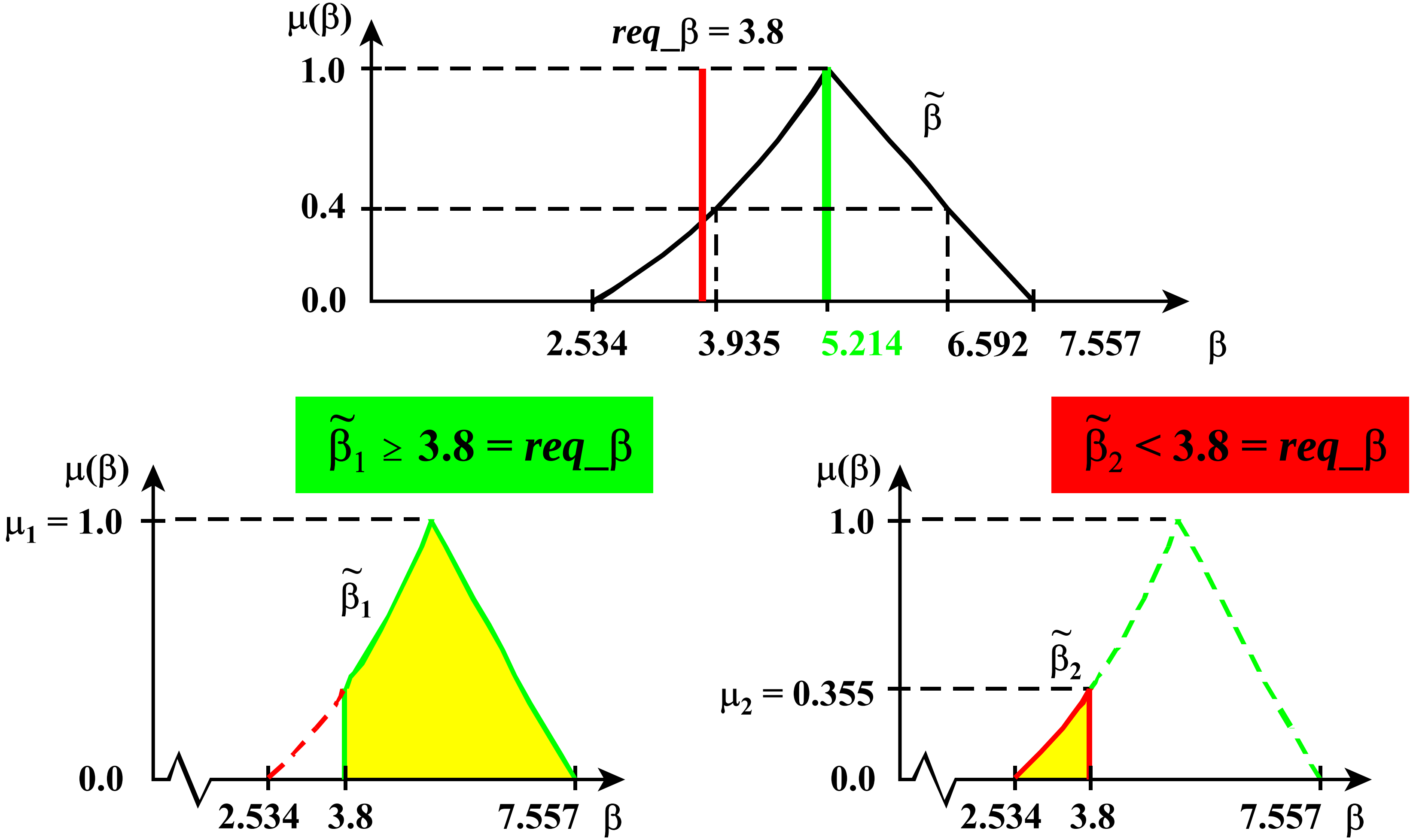
original space of the fuzzy probabilistic basic variables

- **fuzzy probability density function, fuzzy limit state**



FFORM - Analysis II (cont'd)

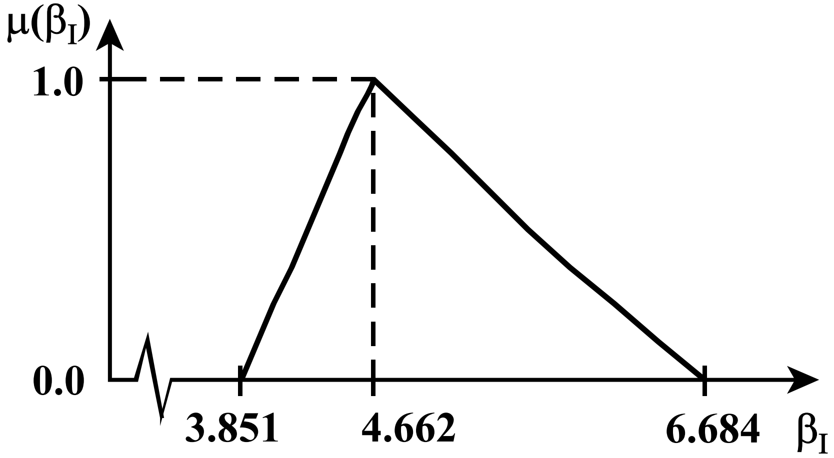
fuzzy reliability index, safety verification

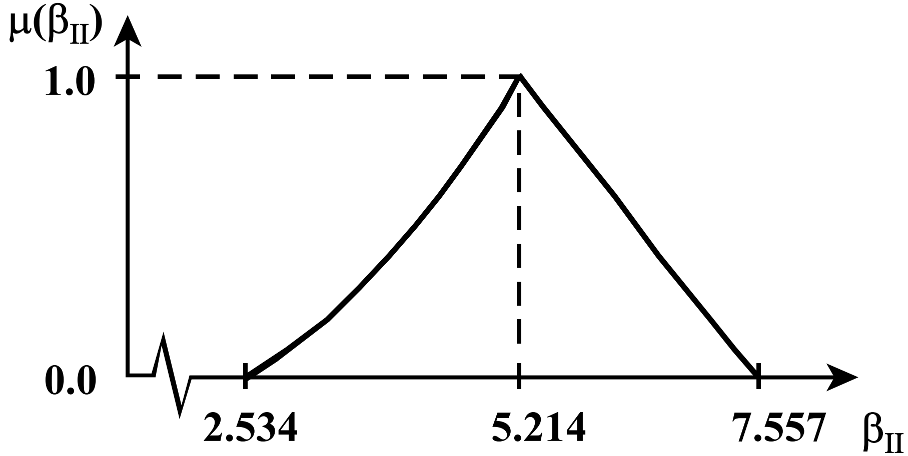


Comparison: Analysis I - Analysis II

analysis I:
fuzzy reliability index β_I

analysis II:
fuzzy reliability index β_{II}





- **modified SHANNON's entropy**

$$\mathbf{H_u}(\tilde{\beta}) = -\mathbf{k} \cdot \int_{\beta=\beta_{\alpha=0,l}}^{\beta=\beta_{\alpha=0,r}} \left[\mu(\beta) \cdot \ln\left(\mu(\beta)\right) + \left(1-\mu(\beta)\right) \cdot \ln\left(1-\mu(\beta)\right)\right] d\beta$$

$$\mathbf{H_u}(\tilde{\beta}_I) = 1.41 \cdot \mathbf{k} < 2.48 \cdot \mathbf{k} = \mathbf{H_u}(\tilde{\beta}_{II})$$

Influence - deterministic fundamental solution

- **load factor ν:**
  extreme value distribution
  ex-max-type I (GUMBEL)

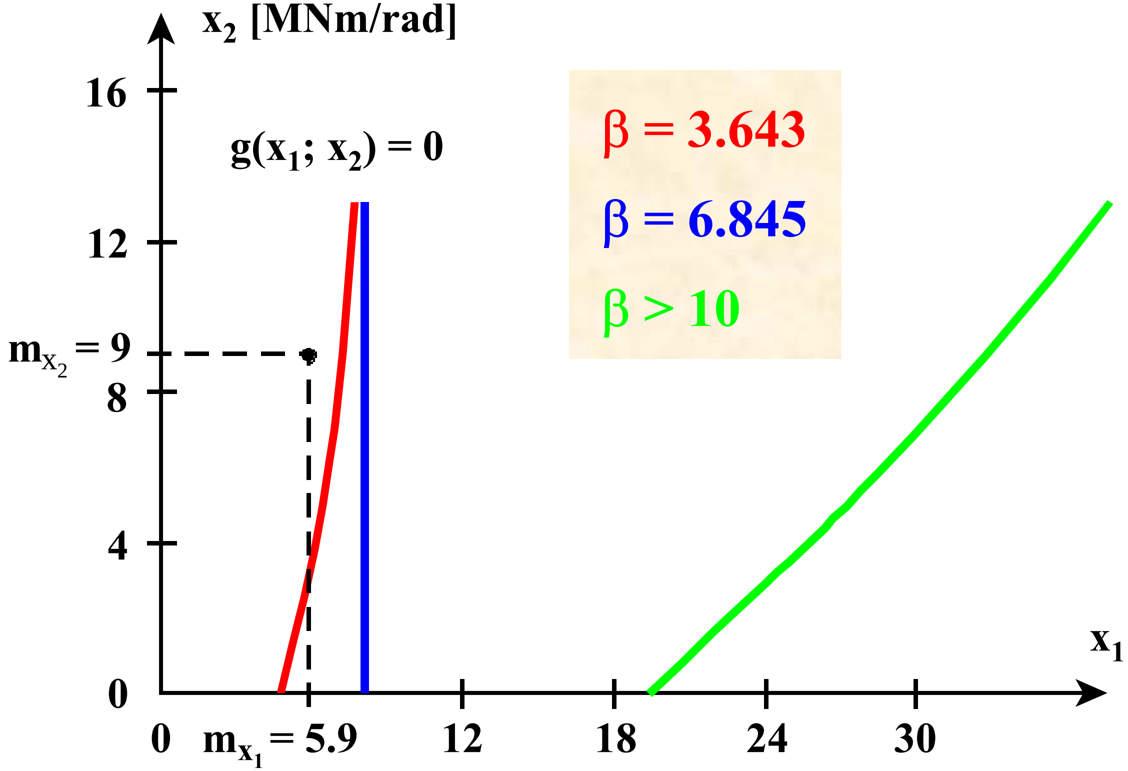
  $m_{x_1} = 5.9$
  $\sigma_{x_1} = 0.11$

- **rotational spring stiffness k_φ:**
  beta distribution

  $x_{min, 2} = 0$ MNm/rad
  $x_{max, 2} = 12$ MNm/rad
  $m_{x_2} = 9.0$ MNm/rad
  $\sigma = 1.35$ MNm/rad

- **computational model**

  geometrically and physically nonlinear (red)
  only physically nonlinear (blue)
  only geometrically nonlinear (green)



Thank you !